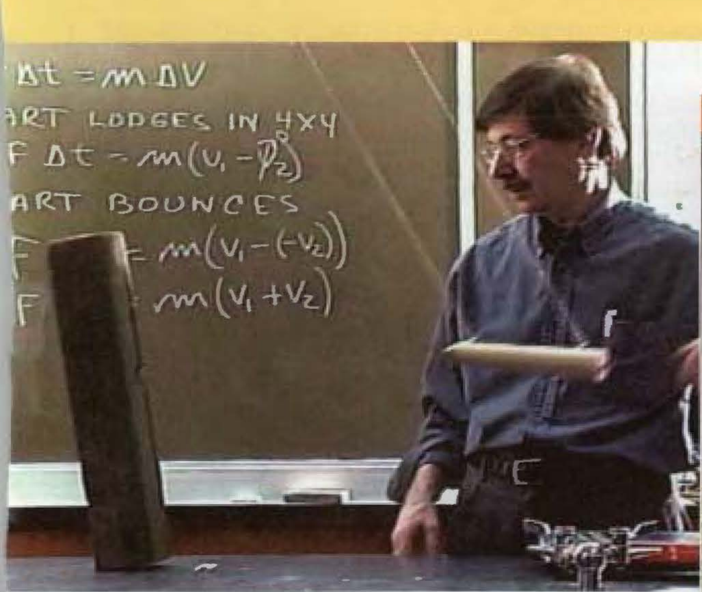
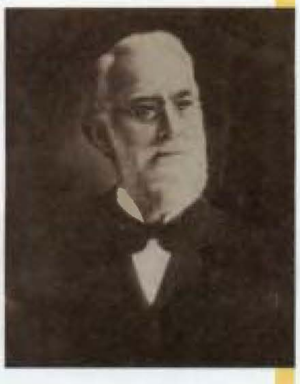
6 Momentum



1 Howie Brand demonstrates the different results when a dart bounces from a wooden block, rather than sticking to it. A bouncing dart produces more impulse, which tips the block. **2** Likewise for the Pelton wheel, where water bouncing from the curved paddles produces more impulse, which imparts more momentum to the wheel. **3** Momentum is mass times speed, as Alex Hewitt shows with his skateboard.

The gold rush that started in 1849 in California brought wealth to many who arrived with picks, shovels, and equipment for gold mining. But mining wasn't the only way to make money in the gold rush. Lester A. Pelton showed up without pick, shovel, or mining equipment and made his fortune by applying some physics (common sense) to the waterwheels used in mining operations at the time. He saw that the low efficiency of waterwheels was due to their flat paddles. Pelton designed a curved paddle with a ridge in the middle that caused the water to make a pair of U-turns upon impact. This produced more force on the paddles, just as more force is required to catch a ball and toss it back than to merely stop the ball. Water made to bounce exerts a greater impulse on the wheel. Pelton patented his idea and ushered in the impulse water turbine, more simply called the Pelton wheel (above). Pelton's story illustrates the fact that physics can indeed enrich your life in more ways than one.

We begin this chapter by examining the concept of momentum and the impulse that causes it to change.



Lester A. Pelton (1829–1908)

FIGURE 6.1
The boulder, unfortunately, has more momentum than the runner.

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Tutorial
Newton's Third Law and Momentum
Video
Definition of Momentum

Momentum

We all know that a heavy truck is harder to stop than a small car moving at the same speed. We state this fact by saying that the truck has more momentum than the car. By **momentum** we mean inertia in motion. More specifically, momentum is defined as the product of the mass of an object and its velocity; that is,

$$\text{Momentum} = \text{mass} \times \text{velocity}$$

Or, in shorthand notation,

$$\text{Momentum} = mv$$

When direction is not an important factor, we can say:

$$\text{Momentum} = \text{mass} \times \text{speed}$$

which we still abbreviate *mv.*

We can see from the definition that a moving object can have a large momentum if either its mass or its velocity is large or if both its mass and its velocity are large. The truck has more momentum than the car moving at the same speed because it has a greater mass. We can see that a huge ship moving at a small speed can have a large momentum, as can a small bullet moving at a high speed. And, of course, a huge object moving at a high speed, such as a massive truck rolling down a steep hill with no brakes, has a huge momentum, whereas the same truck at rest has no momentum at all—because the v term in mv is zero.

FIGURE 6.2
Why are the engines of a supertanker normally cut off 25 km from port? Timing is especially important when changing momentum.

Impulse

If the momentum of an object changes, then either the mass or the velocity or both change. If the mass remains unchanged, as is most often the case, then the velocity changes and acceleration occurs. What produces acceleration? We know the answer is *force.* The greater the force acting on an object, the greater its change in velocity and, hence, the greater its change in momentum.

But something else is important in changing momentum: time—how long a time the force acts. If you apply a brief force to a stalled automobile, you produce a change in its momentum. Apply the same force over an extended period of time, and you produce a greater change in the automobile's momentum. A force sustained for a long time produces more change in momentum than does the same force applied briefly. So, both force and time interval are important in changing momentum.

The quantity *force × time interval* is called **impulse.** In shorthand notation

$$\text{Impulse} = Ft$$

FIGURE 6.3

When you push with the same force for twice the time, you impart twice the impulse and produce twice the change in momentum.

CHECK POINT

M V M

1. Which has more momentum, a 1-ton car moving at 100 km/h or a 2-ton truck moving at 50 km/h?
2. Does a moving object have impulse?
3. Does a moving object have momentum?
4. For the same force, which cannon imparts a greater impulse to a cannonball—a long cannon or a short one?

Check Your Answers

1. Both have the same momentum (1 ton × 100 km/h = 2 ton × 50 km/h).
2. No, impulse is not something an object *has,* like momentum. Impulse is what an object can *provide* or what it can *experience* when it interacts with some other object. An object cannot possess impulse just as it cannot possess force.
3. Yes, but, like velocity, in a relative sense—that is, with respect to a frame of reference, usually Earth's surface. The momentum possessed by a moving object with respect to a stationary point on Earth may be quite different from the momentum it possesses with respect to another moving object.
4. The long cannon will impart a greater impulse because the force acts over a longer time. (A greater impulse produces a greater change in momentum, so a long cannon will impart more speed to a cannonball than a short cannon.)

Timing is important especially when you're changing your momentum.

Impulse Changes Momentum

The greater the impulse exerted on something, the greater will be the change in momentum. The exact relationship is

$$\text{Impulse} = \text{change in momentum}$$

The symbol p is often used to represent momentum.

We can express all terms in this relationship in shorthand notation and introduce the delta symbol Δ (a letter in the Greek alphabet used to denote "change in" or "difference in"):[1]

$$Ft = \Delta(mv)$$

The impulse–momentum relationship helps us to analyze many examples in which forces act and motion changes. Sometimes the impulse can be considered to be the cause of a change of momentum. Sometimes a change of momentum can be considered to be the cause of an impulse. It doesn't matter which way you think about it. The important thing is that impulse and change of momentum are always linked. Here we will consider some ordinary examples in which impulse is related to

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[1] This relationship is derived by rearranging Newton's second law to make the time factor more evident. If we equate the formula for acceleration, $a = F/m$, with what acceleration actually is, $a = \Delta v/\Delta t$, we get $F/m = \Delta v/\Delta t$. From this we derive $F\Delta t = \Delta(mv)$ Calling Δt simply t, the time interval, $Ft = \Delta(mv)$.

FIGURE 6.4

The force of impact on a golf ball varies throughout the duration of impact.

(1) increasing momentum, (2) decreasing momentum over a long time, and (3) decreasing momentum over a short time.

CASE 1: INCREASING MOMENTUM

To increase the momentum of an object, it makes sense to apply the greatest force possible for as long as possible. A golfer teeing off and a baseball player trying for a home run do both of these things when they swing as hard as possible and follow through with their swings. Following through extends the time of contact.

The forces involved in impulses usually vary from instant to instant. For example, a golf club that strikes a ball exerts zero force on the ball until it comes in contact; then the force increases rapidly as the ball is distorted (Figure 6.4). The force then diminishes as the ball comes up to speed and returns to its original shape. So, when we speak of such forces in this chapter, we mean the *average* force.

CASE 2: DECREASING MOMENTUM

If you were in a car that was out of control and you had to choose between hitting a concrete wall or a haystack, you wouldn't have to call on your knowledge of physics to make up your mind. Common sense tells you to choose the haystack. But, knowing the physics helps you to understand *why* hitting a soft object is entirely different than hitting a hard one. In the case of hitting either the wall or the haystack and coming to a stop, it takes the *same* impulse to decrease your momentum to zero. The same impulse does not mean the same amount of force or the same amount of time; rather it means the same *product* of force and time. By hitting the haystack instead of the wall, you extend the *time during which your momentum is brought to zero.* A longer time interval reduces the force and decreases the resulting deceleration. For example, if the time interval is extended 100 times, the force is reduced to a hundredth. Whenever we wish the force to be small, we extend the time of contact. Hence, the padded dashboards and airbags in motor vehicles.

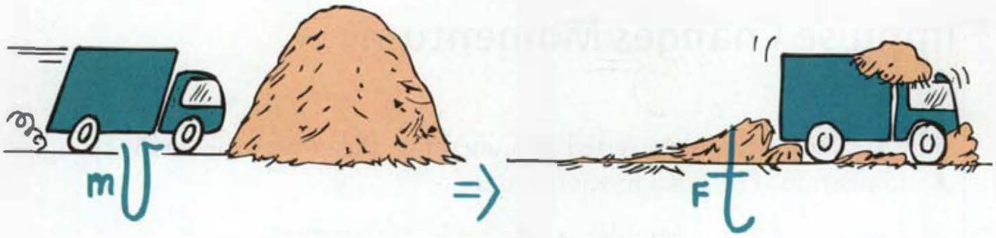

FIGURE 6.5

If the change in momentum occurs over a long time, the hitting force is small.

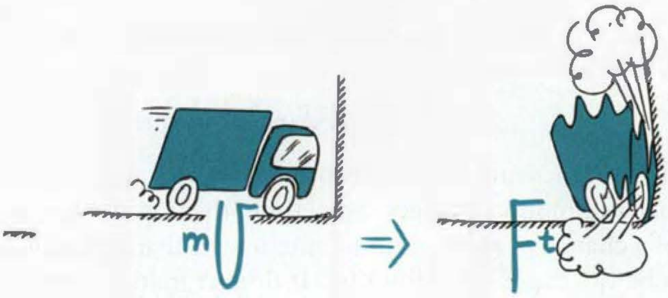

FIGURE 6.6

If the change in momentum occurs over a short time, the hitting force is large.

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Video
Changing Momentum—Follow-Through

When jumping from an elevated position down to the ground, what happens if you keep your legs straight and stiff? Ouch! Instead, you bend your knees when your feet make contact with the ground. By doing so you extend the time during which your momentum decreases by 10 to 20 times that of a stiff-legged, abrupt landing. The resulting force on your bones is reduced by 10 to 20 times. A wrestler thrown to the floor tries to extend his time of impact with the mat by relaxing his muscles and spreading the impact into a series of smaller ones as his foot, knee, hip, ribs, and shoulder successively hit the mat. Of course, falling on a mat is preferable to falling on a solid floor because the mat also increases the time during which the force acts.

The safety net used by circus acrobats is a good example of how to achieve the impulse needed for a safe landing. The safety net reduces the force experienced by a fallen acrobat by substantially increasing the time interval during which the force acts. If you're about to catch a fast baseball with your bare hand, you extend your hand forward so you'll have plenty of room to let your hand move backward after you make contact with the ball. You extend the time of impact and thereby reduce the force of impact. Similarly, a boxer rides or rolls with the punch to reduce the force of impact (Figure 6.8).

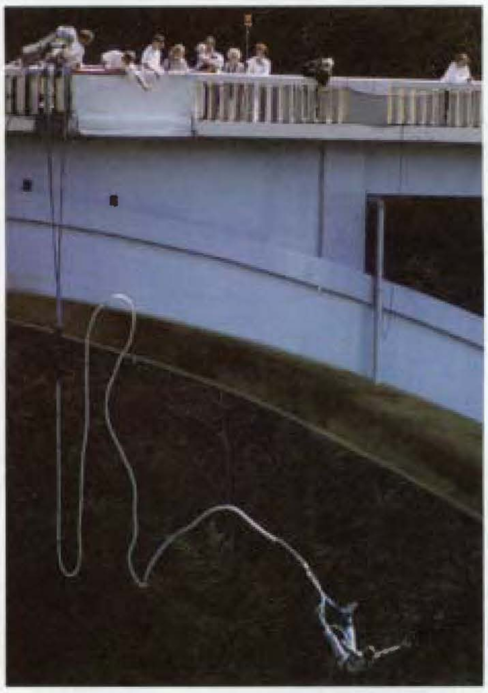

FIGURE 6.7

A large change in momentum over a long time requires a safely small average force.

CASE 3: DECREASING MOMENTUM OVER A SHORT TIME

When boxing, if you move into a punch instead of away, you're in trouble. Likewise, if you catch a high-speed baseball while your hand moves toward the ball instead of away upon contact. Or, when your car is out of control, if you drive it into a concrete wall instead of a haystack, you're really in trouble. In these cases of short impact times, the impact forces are large. Remember that, for an object brought to rest, the impulse is the same, no matter how it is stopped. But, if the time is short, the force will be large.

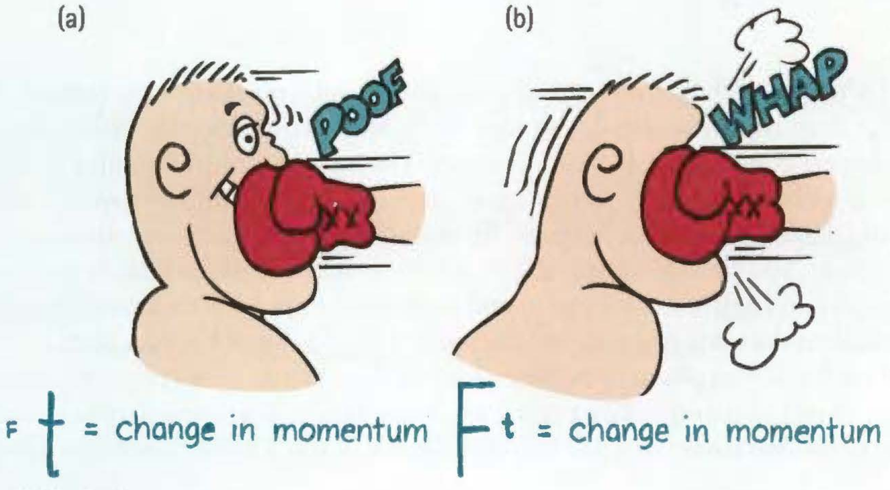

FIGURE 6.8

In both cases, the impulse provided by the boxer's jaw reduces the momentum of the punch. (a) When the boxer moves away (rides with the punch), he extends the time and diminishes the force. (b) If the boxer moves into the glove, the time is reduced and he must withstand a greater force.

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Video
Decreasing Momentum Over a Short Time

The idea of short time of contact explains how a karate expert can split a stack of bricks with the blow of her bare hand (Figure 6.9). She brings her arm and hand swiftly against the bricks with considerable momentum. This momentum is quickly reduced when she delivers an impulse to the bricks. The impulse is the force of her hand against the bricks multiplied by the time during which her hand makes contact with the bricks. By swift execution, she makes the time of contact very brief and correspondingly makes the force of impact huge. If her hand is made to bounce upon impact, the force is even greater.

FIGURE 6.9

Cassy imparts a large impulse to the bricks in a short time and produces a considerable force.

Different forces exerted over different time intervals can produce the same impulse.

Ft or Ft

CHECK POINT

1. If the boxer in Figure 6.8 is able to increase the duration of impact 3 times as long by riding with the punch, by how much will the force of impact be reduced?
2. If the boxer instead moves *into* the punch so as to decrease the duration of impact by half, by how much will the force of impact be increased?
3. A boxer being hit with a punch contrives to extend time for best results, whereas a karate expert delivers a force in a short time for best results. Isn't there a contradiction here?
4. When does impulse equal momentum?

Check Your Answers

1. The force of impact will be only a third of what it would have been if he hadn't pulled back.
2. The force of impact will be 2 times greater than it would have been if he had held his head still. Impacts of this kind account for many knockouts.
3. There is no contradiction because the best results for each are quite different. The best result for the boxer is reduced force, accomplished by maximizing time, and the best result for the karate expert is increased force delivered in minimum time.
4. Generally, impulse equals a *change* in momentum. If the initial momentum of an object is zero when the impulse is applied, then impulse = final momentum. And, if an object is brought to rest, impulse = initial momentum.

Bouncing

A flowerpot dropped onto your head bounces quickly. Ouch! If bouncing took a longer time, as with a safety net, then the force of the bounce would be much smaller.

If a flowerpot falls from a shelf onto your head, you may be in trouble. If it bounces from your head, you may be in more serious trouble. Why? Because impulses are greater when an object bounces. The impulse required to bring an object to a stop and then to "throw it back again" is greater than the impulse required merely to bring the object to a stop. Suppose, for example, that you catch the falling pot with your hands. You provide an impulse to reduce its momentum to zero. If you throw the pot upward again, you have to provide additional impulse. This increased amount of impulse is the same that your head supplies if the flowerpot bounces from it.

The left opening photo at the beginning of this chapter shows physics instructor Howie Brand swinging a dart against a wooden block. When the dart has a nail at its nose, the dart comes to a halt as it sticks to the block. The block remains upright.

FIGURE 6.10

Another view of a Pelton wheel. The curved blades cause water to bounce and make a U-turn, which produces a greater impulse to turn the wheel.

When the nail is removed and the nose of the dart is half of a solid rubber ball, the dart bounces upon contact with the block. The block topples over. The force against the block is greater when bouncing occurs.

The fact that impulses are greater when bouncing occurs was used with great success during the California Gold Rush, as discussed at the beginning of the chapter. Pelton designed a curved paddle that caused the incoming water to bounce upon impact, increasing the impulse on the wheel.

CHECK POINT

1. In reference to Figure 6.9, how does the force that Cassy exerts on the bricks compare with the force exerted on her hand?
2. How will the impulse resulting from the impact differ if her hand bounces back upon striking the bricks?

Check Your Answers

1. In accord with Newton's third law, the forces will be equal. Only the resilience of the human hand and the training she has undergone to toughen her hand allow this feat to be performed without broken bones.
2. The impulse will be greater if her hand bounces from the bricks upon impact. If the time of impact is not correspondingly increased, a greater force is then exerted on the bricks (and her hand!).

Conservation of Momentum

From Newton's second law, you know that to accelerate an object, a net force must be applied to it. This chapter states much the same thing, but in different language. If you wish to change the momentum of an object, exert an impulse on it.

Only an impulse external to a system can change the momentum of the system. Internal forces and impulses won't work. For example, the molecular forces within a baseball have no effect on the momentum of the baseball, just as a push against the dashboard of a car you're sitting in does not affect the momentum of the car. Molecular forces within the baseball and a push on the dashboard are internal forces. They come in balanced pairs that cancel to zero within the object. To change the momentum of the ball or the car, an external push or pull is required. If no external force is present, then no external impulse is present, and no change in momentum is possible.

As another example, consider the cannon being fired in Figure 6.11. The force on the cannonball inside the cannon barrel is equal and opposite to the force causing the cannon to recoil. Since these forces act for the same time, the impulses are also equal and opposite. Recall Newton's third law about action and reaction forces. It applies to impulses, too. These impulses are internal to the system comprising the cannon and the cannonball, so they don't change the momentum of the

Momentum is conserved for all collisions, elastic and inelastic (whenever external forces don't interfere).

fyi

- In Figure 6.11, most of the cannonball's momentum is in speed; most of the recoiling cannon's momentum is in mass. So $mV = {\Large m}v$.

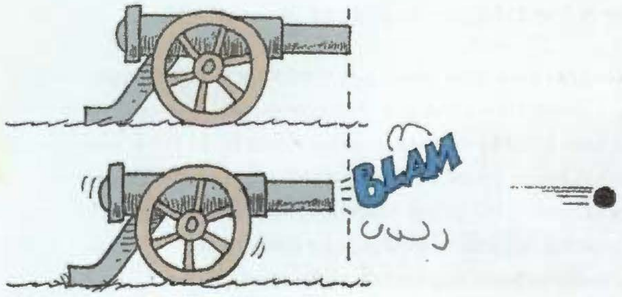


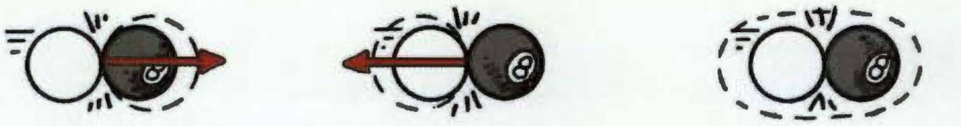
FIGURE 6.11

INTERACTIVE FIGURE

The momentum before firing is zero. After firing, the net momentum is still zero, because the momentum of the cannon is equal and opposite to the momentum of the cannonball.

cannon–cannonball system. Before the firing, the system is at rest and the momentum is zero. After the firing, the net momentum, or total momentum, is *still* zero. Net momentum is neither gained nor lost.

Momentum, like the quantities velocity and force, has both direction and magnitude. It is a *vector quantity.* Like velocity and force, momentum can be cancelled. So, although the cannonball in the preceding example gains momentum when fired and the recoiling cannon gains momentum in the opposite direction, there is no gain in the cannon–cannonball *system.* The momenta (plural form of momentum) of the cannonball and the cannon are equal in magnitude and opposite in direction.[2] Therefore, these momenta cancel to zero for the system as a whole. Since no net external force acts on the system, there is no net impulse on the system and no net change in momentum. You can see that, *if no net force or net impulse acts on a system, the momentum of that system cannot change.*



8-ball system cue-ball system cue-ball + 8-ball system

FIGURE 6.12

A cue ball hits an 8 ball head-on. Consider this event in three systems: (a) An external force acts on the 8-ball system, and its momentum increases. (b) An external force acts on the cue-ball system, and its momentum decreases. (c) No external force acts on the cue-ball + 8-ball system, and momentum is conserved (simply transferred from one part of the system to the other).

When momentum, or any quantity in physics, does not change, we say it is *conserved.* The idea that momentum is conserved when no external force acts is elevated to a central law of mechanics, called the **law of conservation of momentum,** which states:

In the absence of an external force, the momentum of a system remains unchanged.

In any system wherein all forces are internal—as, for example, cars colliding, atomic nuclei undergoing radioactive decay, or stars exploding—the net momentum of the system before and after the event is the same.

Can you see how Newton's laws relate to momentum conservation?

CHECK POINT

1. Newton's second law states that, if no net force is exerted on a system, no acceleration occurs. Does it follow that no change in momentum occurs?
2. Newton's third law states that the force a cannon exerts on a cannonball is equal and opposite to the force the cannonball exerts on the cannon. Does it follow that the *impulse* the cannon exerts on the cannonball is equal and opposite to the *impulse* the cannonball exerts on the cannon?

Check Your Answers

1. Yes, because no acceleration means that no change occurs in velocity or in momentum (mass × velocity). Another line of reasoning is simply that no net force means there is no net impulse and thus no change in momentum.
2. Yes, because the interaction between both occurs during the same *time* interval. Since time is equal and the forces are equal and opposite, the impulses, *Ft*, are also equal and opposite. Impulse is a vector quantity and can be cancelled.

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[2] Here we neglect the momentum of ejected gases from the exploding gunpowder, which can be considerable. Firing a gun with blanks at close range is a definite no-no because of the considerable momentum of ejecting gases. More than one person has been killed by close-range firing of blanks. In 1998, a minister in Jacksonville, Florida, dramatizing his sermon before several hundred parishioners, including his family, shot himself in the head with a blank round from a .357-caliber Magnum. Although no slug emerged from the gun, exhaust gases did—enough to be lethal. So, strictly speaking, the momentum of the bullet + the momentum of the exhaust gases is equal to the opposite momentum of the recoiling gun.

Conservation Laws

A conservation law specifies that certain quantities in a system remain precisely constant, regardless of what changes may occur within the system. It is a law of constancy during change. In this chapter, we see that momentum is unchanged during collisions. We say that momentum is conserved. In the next chapter, we'll learn that energy is conserved as it transforms—the amount of energy in light, for example, transforms completely to thermal energy when the light is absorbed. We'll see, in Chapter 8, that angular momentum is conserved—whatever the rotational motion of a planetary system, its angular momentum remains unchanged so long as it is free of outside influences. In Chapter 22, we'll learn that electric charge is conserved, which means that it can neither be created nor destroyed. When we study nuclear physics, we'll see that these and other conservation laws rule in the submicroscopic world. Conservation laws are a source of deep insights into the simple regularity of nature and are often considered the most fundamental of physical laws. Can you think of things in your own life that remain constant as other things change?

Collisions

Momentum is conserved in collisions—that is, the net momentum of a system of colliding objects is unchanged before, during, and after the collision. This is because the forces that act during the collision are internal forces—forces acting and reacting within the system itself. There is only a redistribution or sharing of whatever momentum exists before the collision. In any collision, we can say

Net momentum before collision = net momentum after collision.

This is true no matter how the objects might be moving before they collide.

When a moving billiard ball makes a head-on collision with another billiard ball at rest, the moving ball comes to rest and the other ball moves with the speed of the colliding ball. We call this an **elastic collision;** ideally, the colliding objects rebound without lasting deformation or the generation of heat (Figure 6.13). But momentum is conserved even when the colliding objects become entangled during the collision. This is an **inelastic collision,** characterized by deformation, or the generation of heat, or both. In a perfectly inelastic collision, both objects stick together. Consider, for example, the case of a freight car moving along a track and colliding with another freight car at rest (Figure 6.14). If the freight cars are of equal mass and are coupled by the collision, can we predict the velocity of the coupled cars after impact?

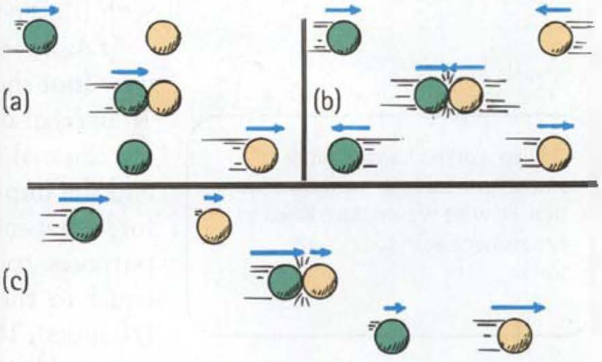


FIGURE 6.13

INTERACTIVE FIGURE

Elastic collisions of equally massive balls. (a) A green ball strikes a yellow ball at rest. (b) A head-on collision. (c) A collision of balls moving in the same direction. In each case, momentum is transferred from one ball to the other.

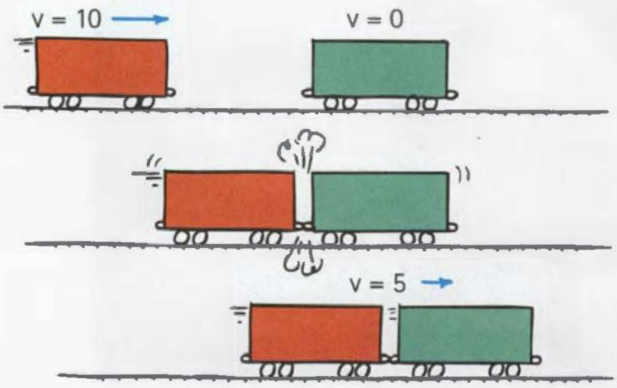


FIGURE 6.14

INTERACTIVE FIGURE

Inelastic collision. The momentum of the freight car on the left is shared with the same-mass freight car on the right after collision.

Suppose the single car is moving at 10 meters per second (m/s), and we consider the mass of each car to be *m*. Then, from the conservation of momentum,

$$(\text{net } mv)_{\text{before}} = (\text{net } mv)_{\text{after}}$$

$$(m \times 10)_{\text{before}} = (2m \times V)_{\text{after}}$$

By simple algebra, $V = 5$ m/s. This makes sense because, since twice as much mass is moving after the collision, the velocity must be half as much as the velocity before collision. Both sides of the equation are then equal.

Note the inelastic collisions shown in Figure 6.15. If A and B are moving with equal momenta in opposite directions (A and B colliding head-on), then one of these is considered to be negative, and the momenta add algebraically to zero. After collision, the coupled wreck remains at the point of impact, with zero momentum.

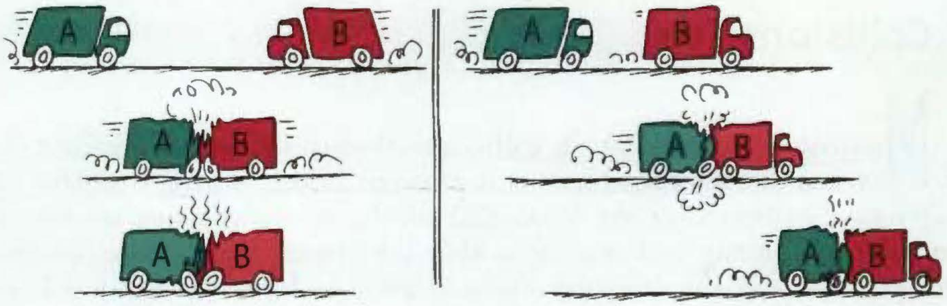


FIGURE 6.15

INTERACTIVE FIGURE

Inelastic collisions. The net momentum of the trucks before and after collision is the same.

If, on the other hand, A and B are moving in the same direction (A catching up with B), the net momentum is simply the addition of their individual momenta.

If A, however, moves east with, say, 10 more units of momentum than B moving west (not shown in the figure), after collision, the coupled wreck moves east with 10 units of momentum. The wreck will finally come to a rest, of course, because of the external force of friction by the ground. The time of impact is short, however, and the impact force of the collision is so much greater than the external friction force that momentum immediately before and after the collision is, for practical purposes, conserved. The net momentum just before the trucks collide (10 units) is equal to the combined momentum of the crumpled trucks just after impact (10 units). The same principle applies to gently docking spacecraft, where friction is entirely absent. Their net momentum just before docking is preserved as their net momentum just after docking.

Galileo worked hard to produce smooth surfaces to minimize friction. How he would have loved to experiment with today's air tracks!

FIGURE 6.16

Will Maynez demonstrates his air track. Blasts of air from tiny holes provide a friction-free surface for the carts to glide upon.

CHECK POINT

Consider the air track in Figure 6.16. Suppose a gliding cart with a mass of 0.5 kg bumps into, and sticks to, a stationary cart that has a mass of 1.5 kg. If the speed of the gliding cart before impact is v_{before}, how fast will the coupled carts glide after collision?

Check Your Answer

According to momentum conservation, the momentum of the 0.5-kg cart before the collision = momentum of both carts stuck together afterwards.

$$0.5v_{\text{before}} = (0.5 + 1.5)\, v_{\text{after}}$$

$$v_{\text{after}} = \frac{0.5\, v_{\text{before}}}{(0.5 + 1.5)} = \frac{0.5\, v_{\text{before}}}{2} = \frac{v_{\text{before}}}{4}$$

This makes sense, because four times as much mass will be moving after the collision, so the coupled carts will glide more slowly. The same momentum means four times the mass glides 1/4 as fast.

For a numerical example of momentum conservation, consider a fish that swims toward and swallows a smaller fish at rest (Figure 6.17). If the larger fish has a mass of 5 kg and swims 1 m/s toward a 1-kg fish, what is the velocity of the larger fish immediately after lunch? Neglect the effects of water resistance.

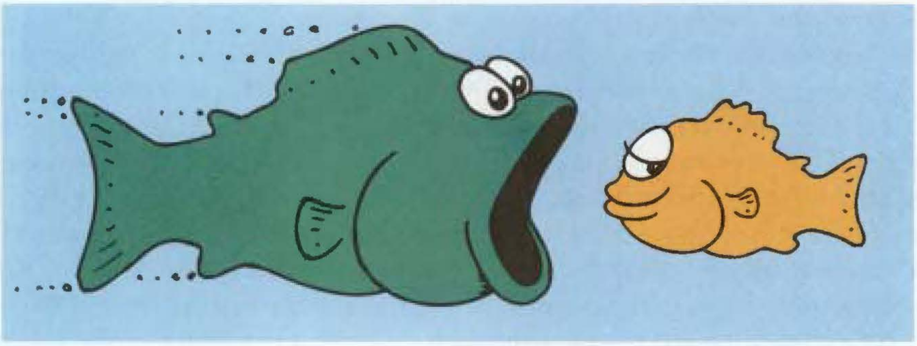


FIGURE 6.17

Two fish make up a system, which has the same momentum just before lunch and just after lunch.

$$\begin{aligned}
\text{Net momentum before lunch} &= \text{net momentum after lunch}\\
(5\text{ kg})(1\text{ m/s}) + (1\text{ kg})(0\text{ m/s}) &= (5\text{ kg} + 1\text{ kg})\, v\\
5\text{ kg}\cdot\text{m/s} &= (6\text{ kg})\, v\\
v &= 5/6\text{ m/s}
\end{aligned}$$

Here we see that the small fish has no momentum before lunch because its velocity is zero. After lunch, the combined mass of both fishes moves at velocity v, which, by simple algebra, is seen to be 5/6 m/s. This velocity is in the same direction as that of the larger fish.

Suppose the small fish in this example is not at rest, but swims toward the left at a velocity of 4 m/s. It swims in a direction opposite that of the larger fish—a negative direction, if the direction of the larger fish is considered positive. In this case,

$$\begin{aligned}
\text{Net momentum before lunch} &= \text{net momentum after lunch}\\
(5\text{ kg})(1\text{ m/s}) + (1\text{ kg})(-4\text{ m/s}) &= (5\text{ kg} + 1\text{ kg})\, v\\
(5\text{ kg}\cdot\text{m/s}) - (4\text{ kg}\cdot\text{m/s}) &= (6\text{ kg})\, v\\
1\text{ kg}\cdot\text{m/s} &= 6\text{ kg}\, v\\
v &= 1/6\text{ m/s}
\end{aligned}$$

Note that the negative momentum of the smaller fish before lunch effectively slows the larger fish after lunch. If the smaller fish were swimming twice as fast, then

$$\begin{aligned}
\text{Net momentum before lunch} &= \text{net momentum after lunch}\\
(5\text{ kg})(1\text{ m/s}) + (1\text{ kg})(-8\text{ m/s}) &= (5\text{ kg} + 1\text{ kg})\, v\\
(5\text{ kg}\cdot\text{m/s}) - (8\text{ kg}\cdot\text{m/s}) &= (6\text{ kg})\, v\\
-3\text{ kg}\cdot\text{m/s} &= 6\text{ kg}\, v\\
v &= -1/2\text{ m/s}
\end{aligned}$$

Here we see the final velocity is $-1/2$ m/s. What is the significance of the minus sign? It means that the final velocity is *opposite* to the initial velocity of the larger fish. After lunch, the two-fish system moves toward the left. We leave as a chapter-end problem finding the initial velocity of the smaller fish to halt the larger fish in its tracks.

More Complicated Collisions

The net momentum remains unchanged in any collision, regardless of the angle between the paths of the colliding objects. Expressing the net momentum when different directions are involved can be achieved with the parallelogram rule of vector addition. We will not treat such complicated cases in great detail here, but will show some simple examples to convey the concept.

Unlike billiard balls after a collision, nuclear particles experience no air drag or other friction and fly on in straight lines without losing speed until hitting another particle or undergoing radioactive decay.

In Figure 6.18, we see a collision between two cars traveling at right angles to each other. Car A has a momentum directed due east, and car B's momentum is directed due north. If their individual momenta are equal in magnitude, then their combined momentum is in a northeasterly direction. This is the direction the coupled cars will travel after collision. We see that, just as the diagonal of a square is not equal to the sum of two of the sides, the magnitude of the resulting momentum will not simply equal the arithmetic sum of the two momenta before collision. Recall the relationship between the diagonal of a square and the length of one of its sides, Figure 5.23 in Chapter 5—the diagonal is $\sqrt{2}$ times the length of the side of a square. So, in this example, the magnitude of the resultant momentum will be equal to $\sqrt{2}$ times the momentum of either vehicle.

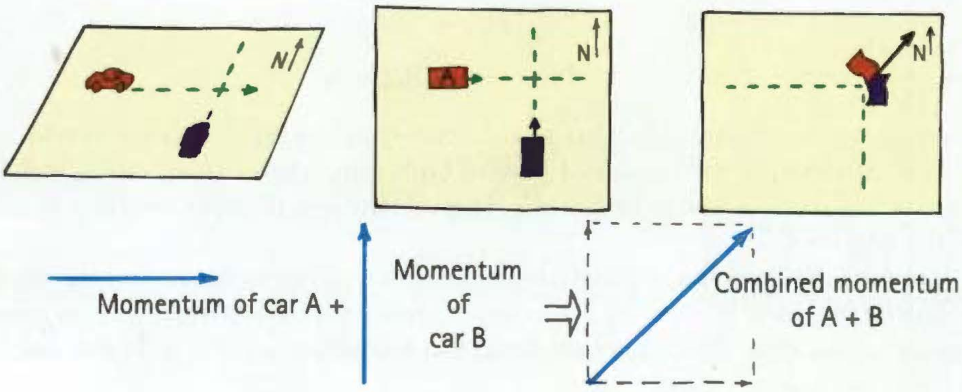


FIGURE 6.18
INTERACTIVE FIGURE
Momentum is a vector quantity.

Figure 6.20 shows a falling Fourth-of-July firecracker exploding into two pieces. The momenta of the fragments combine by vector addition to equal the original momentum of the falling firecracker. Figure 6.19b extends this idea to the microscopic realm, where the tracks of subatomic particles are revealed in a liquid hydrogen bubble chamber.

Whatever the nature of a collision or however complicated it is, the total momentum before, during, and after remains unchanged. This extremely useful law enables us to learn much from collisions without knowing any details about the forces that act in the collision. We will see, in the next chapter, that energy, perhaps in multiple forms, is also conserved. By applying momentum and energy conservation to the collisions of subatomic particles as observed in various detection chambers, we can compute the masses of these tiny particles. We obtain this information by measuring momenta and energy before and after collisions. Remarkably, this achievement is possible without any exact knowledge of the forces that act.

Conservation of momentum and conservation of energy (which we will cover in the next chapter) are the two most powerful tools of mechanics. Applying them yields detailed information that ranges from facts about the interactions of subatomic particles to the structure and motion of entire galaxies.

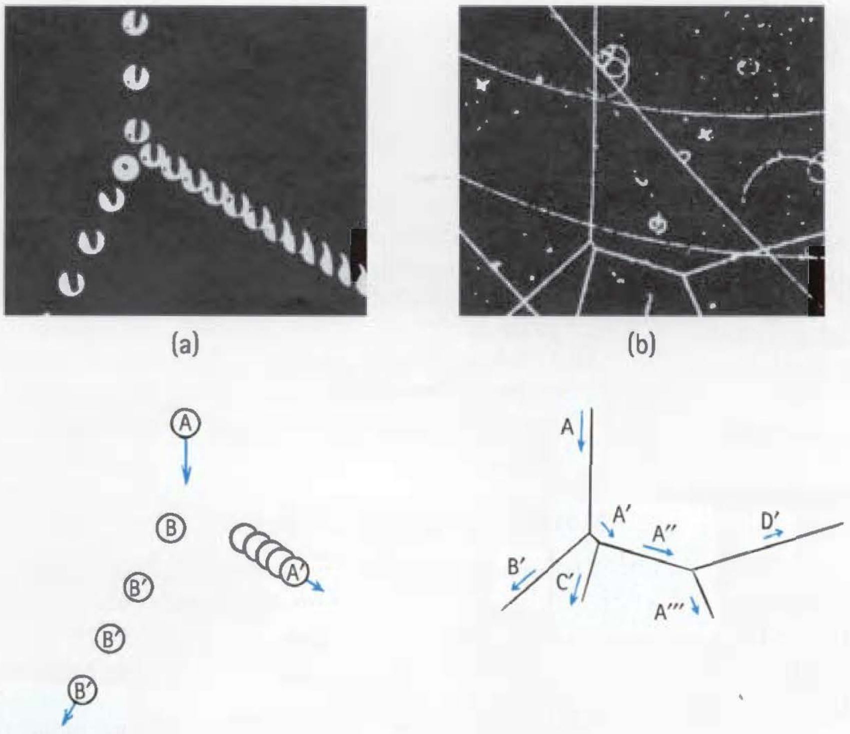


FIGURE 6.19

Momentum is conserved for colliding billiard balls and for colliding nuclear particles in a liquid hydrogen bubble chamber. In (a), billiard ball A strikes billiard ball B, which was initially at rest. In (b), proton A collides successively with protons B, C, and D. The moving protons leave tracks of tiny bubbles.

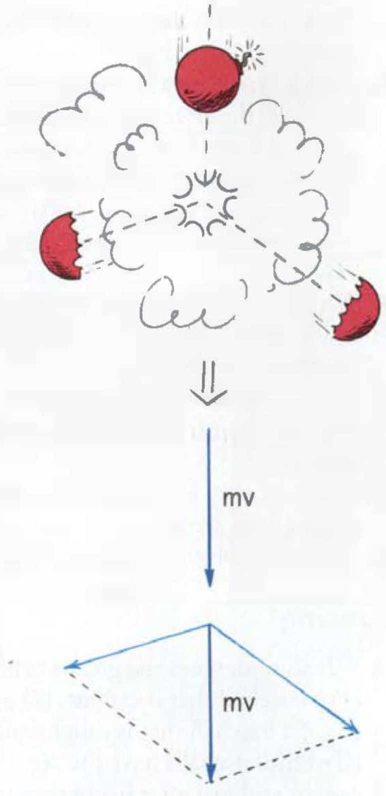


FIGURE 6.20

After the firecracker bursts, the momenta of its fragments add up (by vector addition) to the original momentum.

SUMMARY OF TERMS

Momentum The product of the mass of an object and its velocity.

Impulse The product of the force acting on an object and the time during which it acts.

Relationship of impulse and momentum Impulse is equal to the change in the momentum of the object that the impulse acts upon. In symbol notation,

$$Ft = \Delta mv$$

Law of conservation of momentum In the absence of an external force, the momentum of a system remains unchanged. Hence, the momentum before an event involving only internal forces is equal to the momentum after the event:

$$mv_{(\text{before event})} = mv_{(\text{after event})}$$

Elastic collision A collision in which colliding objects rebound without lasting deformation or the generation of heat.

Inelastic collision A collision in which the colliding objects become distorted, generate heat, and possibly stick together.

REVIEW QUESTIONS

Momentum

1. Which has a greater momentum, a heavy truck at rest or a moving skateboard?

Impulse

2. How does impulse differ from force?
3. What are the two ways to increase impulse?
4. For the same force, why does a long cannon impart more speed to a cannonball than a small cannon?

Impulse Changes Momentum

5. Is the impulse–momentum relationship related to Newton's second law?
6. To impart the greatest momentum to an object, should you exert the largest force possible, extend that force for as long a time as possible, or both? Explain.
7. When you are in the way of a moving object and an impact force is your fate, are you better off decreasing its momentum over a short time or over a long time? Explain.
8. Why is it a good idea to have your hand extended forward when you are getting ready to catch a fast-moving baseball with your bare hand?
9. Why would it be a poor idea to have the back of your hand up against the outfield wall when you catch a long fly ball?
10. In karate, why is a force that is applied for a short time more advantageous?
11. In boxing, why is it advantageous to roll with the punch?

Bouncing

12. Which undergoes the greatest change in momentum: (1) a baseball that is caught, (2) a baseball that is thrown, or (3) a baseball that is caught and then thrown back, if all of the baseballs have the same speed just before being caught and just after being thrown?
13. In the preceding question, in which case is the greatest impulse required?

Conservation of Momentum

14. Can you produce a net impulse on an automobile by sitting inside and pushing on the dashboard? Can the internal forces within a soccer ball produce an impulse on the soccer ball that will change its momentum?
15. Is it correct to say that, if no net impulse is exerted on a system, then no change in the momentum of the system will occur?
16. What does it mean to say that momentum (or any quantity) is *conserved?*
17. When a cannonball is fired, momentum is conserved for the *system* of cannon plus cannonball. Would momentum be conserved for the system if momentum were not a vector quantity? Explain.

Collisions

18. Distinguish between an *elastic collision* and an *inelastic collision.* For which type of collision is momentum conserved?
19. Railroad car A rolls at a certain speed and makes a perfectly elastic collision with car B of the same mass. After the collision, car A is observed to be at rest. How does the speed of car B compare with the initial speed of car A?
20. If the equally massive cars of the previous question stick together ~~after colliding~~ inelastically, how does their speed after the collision compare with the initial speed of car A?

More Complicated Collisions

21. Suppose a ball of putty moving horizontally with 1 kg·m/s of momentum collides and sticks to an identical ball of putty moving vertically with 1 kg·m/s of momentum. Why is their combined momentum not simply the arithmetic sum, 2 kg·m/s?
22. In the preceding question, what is the total momentum of the balls of putty before and after the collision?

PLUG AND CHUG

Momentum = mv

1. What is the momentum of an 8-kg bowling ball rolling at 2 m/s?
2. What is the momentum of a 50-kg carton that slides at 4 m/s across an icy surface?

Impulse = Ft

3. What impulse occurs when an average force of 10 N is exerted on a cart for 2.5 s?
4. What impulse occurs when the same force of 10 N acts on the cart for twice the time?

Impulse = change in momentum: $Ft = \Delta mv$

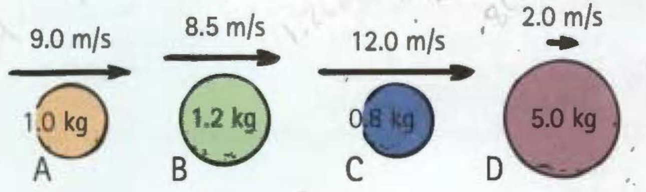
5. What is the impulse on an 8-kg ball rolling at 2 m/s when it bumps into a pillow and stops?
6. How much impulse stops a 50-kg carton sliding at 4 m/s when it meets a rough surface?

Conservation of momentum: $mv_{\text{before}} = mv_{\text{after}}$

7. A 2-kg blob of putty moving at 3 m/s slams into a 2-kg blob of putty at rest. Calculate the speed of the two stuck-together blobs of putty immediately after colliding.
8. Calculate the speed of the two blobs if the one at rest is 4 g.

RANKING

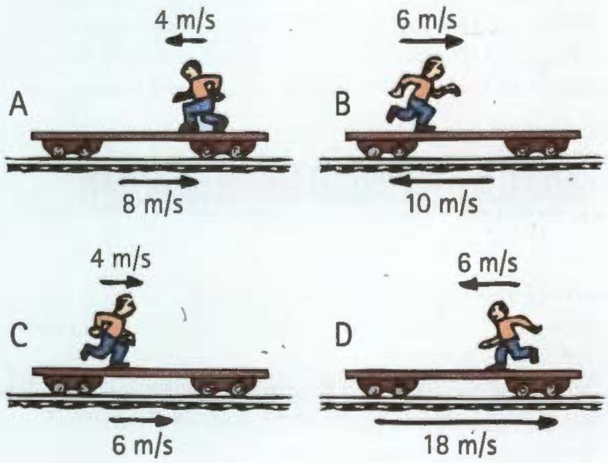
1. The balls have different masses and speeds. Rank the following from greatest to least.



   a. Momentum = m(V), B, D, C, A

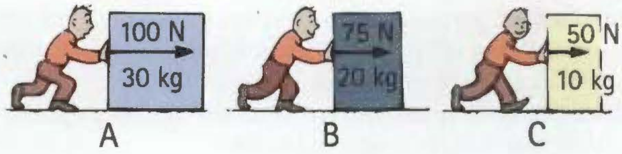
   9, ⓑ The impulses needed to stop the balls

2. Jogging Jake runs along a train flatcar that moves at the velocities shown. In each case, Jake's velocity is given relative to the car. Call direction to the right positive. Rank the following from greatest to least.



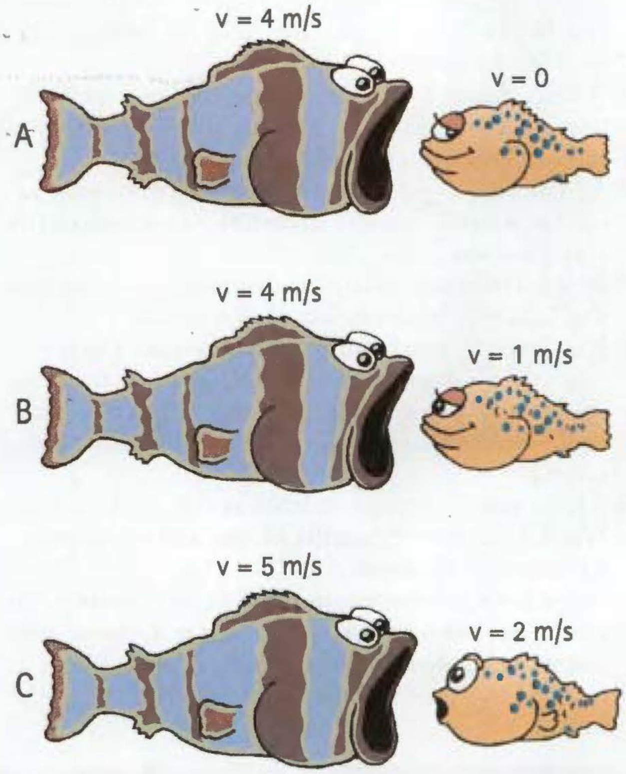
   a. The magnitude of Jake's momentum relative to the flatcar
   b. Jake's momentum relative to an observer at rest on the ground

3. Marshall pushes crates starting from rest across the floor of his classroom for 3 s with a net force as shown. For each crate, rank the following from greatest to least.



   a. Impulse delivered F(t) = A, B, C
   b. Change in momentum
   c. Final speed
   d. Momentum in 3 s

4. A hungry fish is about to have lunch at the speeds shown. Assume the hungry fish has a mass 5 times that of the small fish. Immediately after lunch, for each case, rank from greatest to least the speed of the formerly hungry fish.



PROJECT

When you get a bit ahead in your studies, cut classes some afternoon and visit your local pool or billiards parlor and bone up on momentum conservation. Note that no matter how complicated the collision of balls, the momentum along the line of action of the cue ball before impact is the same as the combined momentum of all the balls along this direction after impact and that the components of momenta perpendicular to this line of action cancel to zero after impact, the same value as before impact in this direction. You'll see both the vector nature of momentum and its conservation more clearly when rotational skidding—"English"—is not imparted to the cue ball. When English is imparted by striking the cue ball off center, rotational momentum, which is also conserved, somewhat complicates analysis. But, regardless of how the cue ball is struck, in the absence of external forces, both linear and rotational momenta are always conserved. Both pool and billiards offer a first-rate exhibition of momentum conservation in action.

EXERCISES

1. When a supertanker is brought to a stop, its engines are typically cut off about 25 km from port. Why is it so difficult to stop or turn a supertanker?
2. In terms of impulse and momentum, why do padded dashboards make automobiles safer?
3. In terms of impulse and momentum, why do air bags in cars reduce the chances of injury in accidents?
4. Why do gymnasts use floor mats that are very thick?
5. In terms of impulse and momentum, why are nylon ropes, which stretch considerably under tension, favored by mountain climbers?
6. Why is it a serious folly for a bungee jumper to use a steel cable rather than an elastic cord?
7. When jumping from a significant height, why is it advantageous to land with your knees bent?
8. A person can survive a feet-first impact at a speed of about 12 m/s (27 mi/h) on concrete; 15 m/s (34 mi/h) on soil; and 34 m/s (76 mi/h) on water. Why the different values for different surfaces?
9. When catching a foul ball at a baseball game, why is it important to extend your bare hands upward so they can move downward as the ball is being caught?
10. Automobiles in past times were manufactured to be as rigid as possible, whereas modern autos are designed to crumple upon impact. Why?
11. In terms of impulse and momentum, why is it important that helicopter blades deflect air downward?
12. It is generally much more difficult to stop a heavy truck than a skateboard when they move at the same speed. State a case in which the moving skateboard could require more stopping force. (Consider relative times.)
13. A lunar vehicle is tested on Earth at a speed of 10 km/h. When it travels as fast on the Moon, is its momentum more, less, or the same?
14. If you throw a raw egg against a wall, you'll break it. But when Peter Hopkinson throws an egg at the same speed into a sagging sheet, it doesn't break. Explain, using concepts from this chapter.

15. Why is it difficult for a firefighter to hold a hose that ejects large amounts of water at a high speed?
16. Would you care to fire a gun that has a bullet 10 times as massive as the gun? Explain.
17. Why are the impulses that colliding objects exert on each other equal and opposite?
18. If a ball is projected upward from the ground with 10 kg•m/s of momentum, what is Earth's momentum of recoil? Why do we not feel this?
19. When an apple falls from a tree and strikes the ground without bouncing, what becomes of its momentum?
20. Why does a baseball catcher's mitt have more padding than a conventional glove?
21. Why do 8-ounce boxing gloves hit harder than 16-ounce gloves?
22. A boxer can punch a heavy bag for more than an hour without tiring but will tire quickly when boxing with an opponent for a few minutes. Why? (*Hint:* When the boxer's fist is aimed at the bag, what supplies the impulse to stop the punches? When the boxer's fist is aimed at the opponent, what or who supplies the impulse to stop the punches that don't connect?)
23. Railroad cars are loosely coupled so that there is a noticeable time delay from the time the first car is moved until the last cars are moved from rest by the locomotive. Discuss the advisability of this loose coupling and slack between cars from the point of view of impulse and momentum.

24. If only an external force can change the velocity of a body, how can the internal force of the brakes bring a moving car to rest?
25. You are at the front of a floating canoe near a dock. You jump, expecting to land on the dock easily. Instead you land in the water. Explain.
26. Explain how a swarm of flying insects can have a net momentum of zero.
27. A fully dressed person is at rest in the middle of a pond on perfectly frictionless ice and must get to shore. How can this be accomplished?
28. If you throw a ball horizontally while standing on roller skates, you roll backward with a momentum that matches that of the ball. Will you roll backward if you go through the motions of throwing the ball, but instead hold on to it? Explain.
29. The examples of the two previous exercises can be explained in terms of momentum conservation and in terms of Newton's third law. Assuming you've answered them in terms of momentum conservation, answer them also in terms of Newton's third law (or vice versa, if you answered already in terms of Newton's third law).
30. In Chapter 5, rocket propulsion was explained in terms of Newton's third law. That is, the force that propels a rocket is from the exhaust gases pushing against the rocket, the reaction to the force the rocket exerts on the exhaust gases. Explain rocket propulsion in terms of momentum conservation.
31. Explain how the conservation of momentum is a consequence of Newton's third law.
32. Go back to Exercise 23 in Chapter 5 and answer it in terms of momentum conservation.

33. If you place a box on an inclined plane, it gains momentum as it slides down. What is responsible for this change in momentum?

34. Your friend says that the law of momentum conservation is violated when a ball rolls down a hill and gains momentum. What do you say?

35. What is meant by a system, and how is it related to the conservation of momentum?

36. If you toss a ball upward, is the momentum of the moving ball conserved? Is the momentum of the system consisting of ball + Earth conserved? Explain your answers.

37. The momentum of an apple falling to the ground is not conserved because the external force of gravity acts on it. But momentum is conserved in a larger system. Explain.

38. Drop a stone from the top of a high cliff. Identify the system wherein the net momentum is zero as the stone falls.

39. A car hurtles off a cliff and crashes on the canyon floor below. Identify the system wherein the net momentum is zero during the crash.

40. Bronco dives from a hovering helicopter and finds his momentum increasing. Does this violate the conservation of momentum? Explain.

41. Which exerts the greater impulse on a steel plate—machine gun bullets that bounce from the plate, or the same bullets squashing and sticking to the plate?

42. An ice sailcraft is stalled on a frozen lake on a windless day. The skipper sets up a fan as shown. If all the wind bounces backward from the sail, will the craft be set in motion? If so, in what direction?

43. Will your answer to the preceding exercise be different if the air is brought to a halt by the sail without bouncing?

44. Discuss the advisability of simply removing the sail in the preceding exercises.

45. As you toss a ball upward, is there a change in the normal force on your feet? Is there a change when you catch the ball? (Think of doing this while standing on a bathroom scale.)

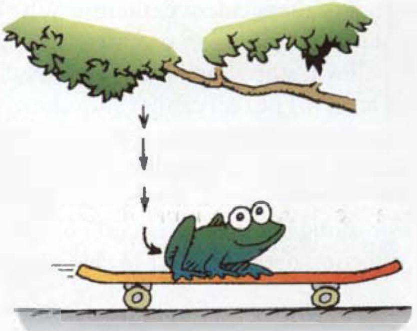
46. When you are traveling in your car at highway speed, the momentum of a bug is suddenly changed as it splatters onto your windshield. Compared with the change in momentum of the bug, by how much does the momentum of your car change?

47. If a tennis ball and a bowling ball collide in midair, does each undergo the same amount of momentum change? Defend your answer.

48. If a Mack truck and a MiniCooper have a head-on collision, which vehicle will experience the greater force of impact? The greater impulse? The greater change in momentum? The greater deceleration?

49. Would a head-on collision between two cars be more damaging to the occupants if the cars stuck together or if the cars rebounded upon impact?

50. Freddy Frog drops vertically from a tree onto a horizontally moving skateboard. The skateboard slows. Give two reasons for this, one in terms of a horizontal friction force between Freddy's feet and the skateboard, and one in terms of momentum conservation.

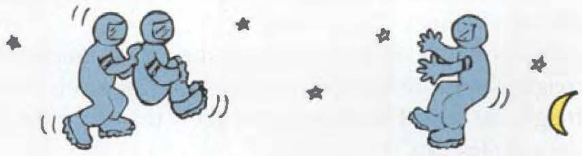


51. A 0.5-kg cart on an air track moves 1.0 m/s to the right, heading toward a 0.8-kg cart moving to the left at 1.2 m/s. What is the direction of the two-cart system's momentum?

52. In a movie, the hero jumps straight down from a bridge onto a small boat that continues to move with no change in velocity. What physics is being violated here?

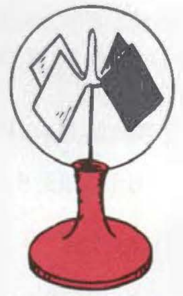
53. To throw a ball, do you exert an impulse on it? Do you exert an impulse to catch it at the same speed? About how much impulse do you exert, in comparison, if you catch it and immediately throw it back again? (Imagine yourself on a skateboard.)

54. Suppose that there are three astronauts outside a spaceship and that they decide to play catch. All the astronauts weigh the same on Earth and are equally strong. The first astronaut throws the second one toward the third one and the game begins. Describe the motion of the astronauts as the game proceeds. How long will the game last?



55. In reference to Figure 6.9, how will the impulse at impact differ if Cassy's hand bounces back upon striking the bricks? In any case, how does the force exerted on the bricks compare to the force exerted on her hand?

56. Light possesses momentum. This can be demonstrated with a radiometer, shown in the sketch. Metal vanes painted black on one side and white on the other are free to rotate around the point of a needle mounted in a vacuum. When light is incident on the black surface, it is absorbed; when light is incident upon the white surface, it is reflected. Upon which surface is the impulse of incident light greater, and which way will the vanes rotate? (They rotate in the opposite direction in the more common radiometers in which air is present in the glass chamber; your instructor may tell you why.)



57. A deuteron is a nuclear particle of unique mass made up of one proton and one neutron. Suppose that a deuteron is accelerated up to a certain very high speed in a cyclotron and directed into an observation chamber, where it collides with and sticks to a target particle that is initially at rest and then is observed to move at exactly half the speed of the incident deuteron. Why do the observers state that the target particle is itself a deuteron?

58. A billiard ball will stop short when it collides head-on with a ball at rest. The ball cannot stop short, however, if the collision is not exactly head-on—that is, if the second ball moves at an angle to the path of the first. Do you know why? (*Hint:* Consider momentum before and after the collision along the initial direction of the first ball and also in a direction perpendicular to this initial direction.)

59. When a stationary uranium nucleus undergoes fission, it breaks into two unequal chunks that fly apart. What can you conclude about the momenta of the chunks? What can you conclude about the relative speeds of the chunks?

60. You have a friend who says that after a golf ball collides with a bowling ball at rest, although the speed gained by the bowling ball is very small, its momentum exceeds the initial momentum of the golf ball. Your friend further asserts this is related to the "negative" momentum of the golf ball after collision. Another friend says this is hogwash—that momentum conservation would be violated. Which friend do you agree with?

PROBLEMS

1. When bowling, your physics buddy asks how much impulse is needed to stop a 10-kg bowling ball moving at 6 m/s. What is your answer?

2. Joanne drives her car with a mass of 1000 kg at a speed of 20 m/s. Show that to bring her car to a halt in 10 s road friction must exert a force of 2000 N on the car.

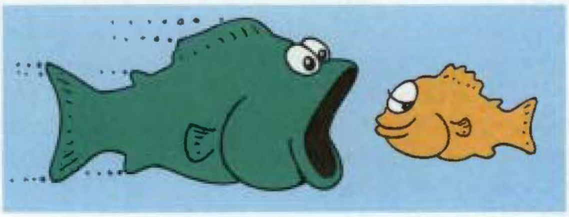
3. A car carrying a 75-kg test dummy crashes into a wall at 25 m/s and is brought to rest in 0.1 s. Show that the average force exerted by the seat belt on the dummy is 18,750 N.

4. Judy (mass 40 kg), standing on slippery ice, catches her leaping dog (mass 15 kg) moving horizontally at 3.0 m/s. Show that the speed of Judy and her dog after the catch is 0.8 m/s.

5. A 2-kg ball of putty moving to the right has a head-on inelastic collision with a 1-kg putty ball moving to the left. If the combined blob doesn't move just after the collision, what can you conclude about the relative speeds of the balls before they collided?

6. A railroad diesel engine weighs four times as much as a freight car. If the diesel engine coasts at 5 km/h into a freight car that is initially at rest, show that the speed of the coupled cars is 4 km/h.

7. A 5-kg fish swimming 1 m/s swallows an absentminded 1-kg fish swimming toward it at a speed that brings both fish to a halt immediately after lunch. Show that the speed of the approaching smaller fish before lunch must have been 5 m/s.



8. Comic-strip hero Superman meets an asteroid in outer space and hurls it at 800 m/s, as fast as a bullet. The asteroid is a thousand times more massive than Superman. In the strip, Superman is seen at rest after the throw. Taking physics into account, what would be his recoil velocity?

9. Two automobiles, each of mass 1000 kg, are moving at the same speed, 20 m/s, when they collide and stick together. In what direction and at what speed does the wreckage move (a) if one car was driving north and one south; (b) if one car was driving north and one east (as shown in Figure 6.18)?

•10. An ostrich egg of mass m is tossed at a speed v into a sagging bed sheet and is brought to rest in a time t.
   a. Show that the force acting on the egg when it hits the sheet is mv/t.
   b. If the mass of the egg is 1 kg, its initial speed is 2 m/s, and the time to stop is 0.2 s, show that the average force on the egg is 10 N.

CHAPTER 6 ONLINE RESOURCES

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- 6.11, 6.13, 6.14, 6.15, 6.18

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- Changing Momentum: Follow-through
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