Mathematics Review for PHYSICS

Introduction

In a sense, mathematics is the language of physics. Since the goal or physics is to describe the world quantitatively (usually), mathematics is the natural language. Even when only qualitative descriptions are needed, the concepts are frequently couched in the language of mathematics. It is necessary to be conversant with simple mathematics to successfully understand physics.

The idea that physics consists of "formulas" from which one calculates results is totally incorrect. Physics requires attaching a physical meaning to the symbols, developing a "feeling" for these quantities, and some intuition is usually required in the solution of problems. The problems you will be doing during this course are similar to the "word" problems you have done in some of your math courses. Only elementary mathematics is required: algebra, geometry, and trigonometry. While all students in this course will have had courses in these subjects, frequently these skills have not been exercised since the courses were taken. During the first few laboratory sessions the concepts most used in elementary physics will be reviewed, and you will work problems, which should help you to overcome difficulties generated by being "stale" in mathematics.

Algebra

A first requirement is that you be able to carry out simple algebraic manipulations and rearrangements with ease, and to attach physical meaning to the variables.

The algebraic rules you will need are in the "summary of math review". These exercises are designed to give you practice in the kind or manipulations and thinking required for physics.

A. Evaluate the following expressions for a = 3, b = -2, c = 4, d = -1, x = 5, and y = -3.

1. $a - 3x - c$
2. $a + 2b - 3d + y$
3. $2c + x - 2b + c$
4. $\dfrac{5 + y}{2d - y}$
5. $\dfrac{(4xy - 3a)d}{2ac + 2y}$
6. $\dfrac{(2c - 3d)(2a - 2c)}{(3c + 8a)(c - d)}$

B. Add the following expressions;

1. (2a + 7b - 15c) + (6a+ 2b + 6c) + (2a + 3b + 4c) - (2a- 3b+ 6c)
2. (3x- 4y + 7z + 4c) + (-2x + 4y - 8z - 2c) - (3x -7z - 3c)

C. Carry out the following multiplications and raising to a power:

1. (3x - 2y) (6x + 3y)
2. (2a + 3b) (a - 2b + c)
3. $(2x - 3y)^2$

D. Solve the following equations for the unknown variable:

1. $4x + 7 = 0$
2. $3(3y - 4) = 6y$
3. $3(3y - 4) = 2(4y - 8)$
4. $0.7x - 1.2 = 0.4(2x - 1)$
5. $4 - \dfrac{3}{x} = 6 - \dfrac{5}{x}$
6. $\dfrac{3}{q} + \dfrac{2}{q} - \dfrac{8}{q} = 11$

E. "Word" problems.

Solving such problems is very similar to much of the problem solving in physics. They require a certain amount of creative thought and a clear understanding of the representation of physical quantities in algebraic form. No "recipe" or rigid set of rules can be given which will solve all such problems, but certain guidelines can be helpful:

1. Read the problem carefully, frequently several readings will be required. Identify the given facts and the unknown quantity to be determined.

2. Introduce a letter to represent the unknown quantity. Words frequently used, but certainly not always, such as "what", "find", "how much", "how far", "when", etc. will help to identify the unknown quantity.

3. If appropriate, draw a picture and label it.

4. Make a list of the pertinent facts and relationships involving the unknown quantity.

5. Using step 4 and rereading the problem, write an equation describing exactly what is stated in words.

6. Solve the equation produced in step 5.

7. Look at your result to see that it is reasonable.

8. Most important, don't get discouraged! It takes considerable effort to become proficient at solving physics problems.

Example: Suppose a student has test scores of 62 and 92. What score must the student make on a third test to have an average of 80?

Let x be the third test.

$$\text{Then the average} = 80 = \frac{62+92+x}{3} \text{ (by definition of the average).}$$

240 = 62 + 92 + x

x = 240 - 154 = 86

Sample:

A store is having a sale discounting all items by 20%. What was the original cost of an item that costs $28 on sale?

Let x be the original cost.
Then x - 0.2x = $28; 0.8x = $28, x = $35

Exercises

1. A radiator contains 6 quarts of a mixture of antifreeze and water. 40% of this mixture is antifreeze. How much of this mixture should be drained and replaced with pure antifreeze so that the mixture will be 60% antifreeze?

2. A car (Car A) leaves a gas station and travels along a straight road 200 miles long at a uniform speed of 40 miles per hour. A second car (Car B) leaves the station 1/2 hour later and travels along the same road at 55 miles per hour. At what time will Car B overtake Car A?

3. How much water must be evaporated from 500 grams of a 10% salt solution to obtain a 15% salt solution?

4. A boy can mow a lawn in 90 minutes and his sister can mow the same lawn in 60 minutes. How long will it take for both mowing at the same time to mow the lawn?

5. The relationship between the Fahrenheit and Celsius temperature scales is:

$$^{\circ}C = \frac{5}{9}(^{\circ}F - 32)$$

   Find the ^{o}C and ^{o}F when the celsius reading (^{o}C) is twice the Fahrenheit reading (^{o}F).

F. Exponents, radicals, and logarithms

Remember a^n means multiply a by itself n times; a^3 = a x a x a and the algebraic rules of exponents are:

$a^m a^n = a^{m+n}$ $(a^m)^n = a^{mn}$

$(ab)^n = a^n b^n$ $(a/b)^n = a^n/b^n$

Thus $a^4/a^6 = a^{-2}$

Usually very small or very large numbers are expressed as powers (or exponents) of 10. Thus $13,250 would be expressed as $1.325 x 10^4. The usual rules of exponents apply, so $(2 \times 10^7)/(4 \times 10^{-3}) = 0.5 \times 10^{10}$; $2 \times 10^7 \times 4 \times 10^{-3} = 8 \times 10^4$.

Exercises

1. $a^3 a^2 =$

2. $b^3 b^{-2} =$

3. $\frac{b^3}{b^{-2}} =$

4. $10^6 \times 10^{-2} =$

5. $\frac{10^6}{10^{-2}} =$

6. $3\times 10^8 \times 2\times 10^3 =$

7. $(4a^3)(3a^3) =$

8. $\frac{5^{-2}}{2^4} =$

9. $\frac{4b^2}{2b^{-4}} =$

10. $\frac{2\times 10^{-5} \times 10^3}{2\times 10^4} =$

11. Express 121,200 in exponential (powers of 10) notation. Express the earth sun distance in kilometers in powers of 10 notation.

12. Express the radius of the hydrogen atom in powers of 10 notation.

Radicals and fractional powers

The equivalent notation $\sqrt{a} = a^{1/2}$, $1/\sqrt{a} = a^{-1/2}$, $\sqrt[3]{a} = a^{1/3}$ etc. is frequently used. The same laws of multiplication and division are valid for fractional exponents as for integral exponents:
$(a^{-2})\ (a^{3/2}) = a$

Exercises

Perform the multiplication indicated leaving answers in exponential form:

1. $(6a^{1/2})\ (b^{3/2})\ (2a^{1/4})\ b^0$

2. $[c^{2/5}\ (2c^{-1/6}\ d^{1/3})]^3$

$$(1)\quad 2x+3y=-10 \qquad (2)\quad \frac{2x}{y}=4$$

$$from\,(2)\; y=\frac{2x}{4}=\frac{x}{2}$$

$$substituting \quad in\,(1)$$

$$2x+\frac{3x}{2}=-10,\; x=-\frac{10}{\frac{7}{2}}=-\frac{20}{7};\; y=\frac{x}{2}=-\frac{20}{14}=-\frac{10}{7}.$$

Express the following in powers of 10 notation:

3. 312 million

4. 0.32 millionths

5. 2.34 million times two hundred thousand divided by 21 millionths

G. Simultaneous linear equations

Frequently a problem involves two or more unknowns and a set of conditions which require the use of two or more equations to determine the unknowns. In Physics 2053 and 2054 you must become proficient in solving 2 equations with 2 unknowns e.g.,

1. 2x + 3y = 10
2. 4x - 2y = 6

To solve these equations solve (1) or (2) for y (or x), substitute into the other equation, thus generating an equation in a single variable. Then solve this equation in the usual way.

From (2): y = 2x - 3 (3)

Substitute into (1); 2x + 3 (2x - 3) = 10

2x + 6x - 9 = 10

+ 8x = 19

$$x = +\frac{19}{8}$$

Substitute $x = +\frac{19}{8}$ *into (3) and solve for* y

$$y = 2 \bullet \frac{19}{8} - 3 = +\frac{19}{4} - 3 = +\frac{7}{4} = 1\frac{3}{4}.$$

Another kind of simultaneous equations which arises is of the form:

Exercises

Solve the following equations for x and y

1. 3x -7y = 13
   2x + 5y = -1

2. 4x + 4y = 1
   x + 6y = 3

3. $\frac{2x}{3} - \frac{y}{2} = 0$
   $\frac{x}{2} - \frac{5y}{8} = -1$

4. Two bank accounts total $8,000. If $300 is taken from one account and put in the other, each will have the same amount. How much is there in each account?

5. 12 yd of cotton cloth and 10 yd of nylon cloth cost $36.3 yd of cotton and 5 yd of nylon cost $15. What is the cost yard of each material?

6. The boiling point of water is 212 ^{o}F or 100 ^{o}C. The freezing point is 32 ^{o}F or 0 ^{o}C. If the equation relating ^{o}F, to ^{o}C is of the form ^{o}F = a ^{o}C + b, find a and b.

7. An airplane flies 560 miles with a tail wind in 2 hr 20 min. It takes 3 hours to fly against the headwind. Find the speed of the airplane in still air and the speed of the wind.

H. Linear equations and their graphical representation

A linear equation relates two variables which are proportional to each other, and has the form y = mx + b; if y = 3x - 2 then m = 3, b = -2

where b is the intercept with the y axis (*i.e.*, for x = 0).

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

1. Plot y vs. x for the following data. Find m and b from this plot.

x	y
0	-3
1	-1
2	+1
3	+3
4	+5
5	+7

2. Put the equation 3y = 4x + 6 into the form y = mx + b. Find m and b.

Quadratic equations

The quadratic equation sometimes occurs in solving physics 2053 and 2054 problems.

The equation is of the form:

$$ax^2 + bx + c = 0$$

Any algebraic equation involving the square of a variable and linear (first power) terms in the variable may be put in this form by algebraic manipulation.

The solution of this equation is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Note that there are generally two solutions to the equation corresponding to the + or - sign in front of the square root sign. Frequently only one of the solutions will have physical meaning. For example, if solving for a time, the root corresponding to a negative time would not have physical meaning. Also, the solution may be imaginary ($4ac > b^2$).

While such solutions may be physically meaningful in more advanced physics, only real solutions are meaningful in Physics 2053-2054. If your solution is imaginary, check your work!

Exercises

Solve for x:

1. $6x^2 + 11x - 10 = 0$

2. $4x^2 + 29x + 30 = 0$

3. $2x^2 - x - 3 = 0$

"Word Problems"

4. A piece of wire 100 inches long is cut into two pieces and each bent into the shape of a square. If the sum of the areas of the two squares is 400 in^2, find the length of each piece of wire.

5. The speed of the current in a river is 5 miles per hour. It takes 40 minutes longer for a girl to paddle a canoe 1.2 miles upstream than the same distance downstream. What is her speed in stillwater?

6. Find the length of the side of a square if the diagonal is 3 ft. longer than a side.

I. Logarithms

If $a^\mu = b$, $\mu = \log_a b$, where a is called the base of the logarithm.

Thus $\log_2 8 = 3$ $(2^3 = 8)$
$\log_{10} 100 = 2$ $(10^2 = 100)$
$\log_{10} 10^6 = 6$

Laws of logarithms

$\log_a u\,v = \log_a u + \log_a v$
$\log_a u/v = \log_a u - \log_a v$

$\log_a (u^c) = c \log_a u$

The use of logarithms can be useful to linearize an equation of the form $y = ax^z$.

From the third law of logarithms:

$\log_{10}y = z \log_{10}x + \log_{10}a$

This equation has the form

$y = mx + b$

So if one plots $\log_{10}y$ vs. $\log_{10}x$ for a physical system obeying $y = ax^z$ then the plot will be linear, and z can be determined from the slope and a from the intercept. You will carry out an experiment later in the semester where this will be required.

A number of physical laws can be described by an exponential of e (=2.71828), (e.g., radioactive decay, absorption of radiation as a function of thickness of the absorber).

Then $y = ae^x$

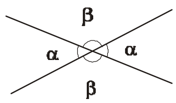
and $\log y = x \log e + \log a$

Thus if log y is plotted vs. x, the result is again or linear form and x and a can be determined from the slope and intercept. You will study and use these logarithm forms in some of your lab experiments.

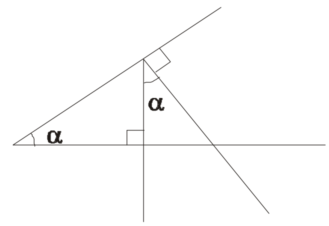
Geometry

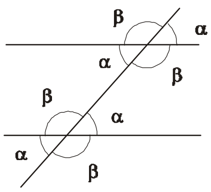
Geometrical relations and properties are necessary to solve many kinds of problems which arise in Physics 2053 and 2054. Problems involving spatial analysis should always be clearly sketched so that the relationship can be more easily visualized.

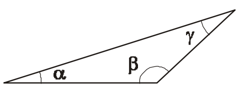
Relations between angles which are commonly used are:



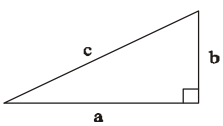
$\alpha + \beta = 180^{o}$







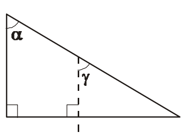
$\alpha + \beta + \gamma = 180^{o} = \pi$ radians



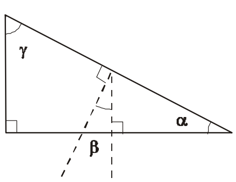
$a^2 + b^2 = c^2$

If two triangles have two internal angles which are common, the third angle must be common, since the sum of the internal angles is 180^{o}. If two triangles of different dimensions are similar, the corresponding sides are proportional to each other.

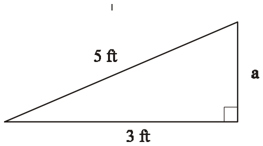
Exercises

1. 

If $\alpha = 20^{o}$ what is the angle of γ?

2. 

If $\alpha = 20^{o}$ what is the β? What is γ?

3. 

What is the length of side a for the right triangle?

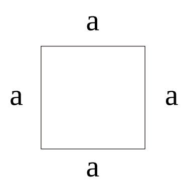
4. A man walks 2 miles east, then 3 miles north. He walks back along the shortest possible distance. Sketch this walk. What is the total distance he walks?

5. A girl stands 40 ft. from a tree. With her eye at ground level she sights along a yard stick and finds the top of the yard stick coincides with the top of the tree when the bottom of the yard stick is 4 feet from her eye. Sketch this situation and find the height of the tree.

It is frequently required to find the distance around a geometric object (perimeter), its area, or its volume in order to solve some physical problems.

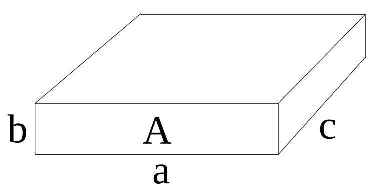
Some of the relations you will need are:

Square:



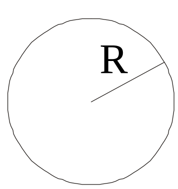
Perimeter	area	volume of cube
4a	a^2	a^3

Rectangular parallelopiped:



Perimeter of face A = 2a + 2b
Area of face A = a x b
Volume = a x b x c

Circle:



Circumference = $2\pi R = \pi D$
Area = πR^2

Sphere: Surface area = $4\pi R^2$

Volume = $\frac{4}{3}\pi R^3$

Exercises

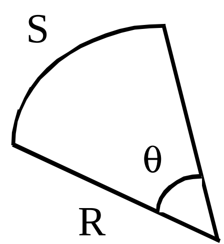
1. Find the area of a rectangle which has sides 3 cm and 6 cm. What would the length of the third side of a box if this rectangle is one face and the volume of the box is 90 cm^3?

2. What is the radius of a circle if the circumference is 10 cm? What is its area?

3. What is the volume of a cube if the sides are 6 cm long? What is the radius of a sphere which has this same volume?

Radian Measure

Frequently it is useful to measure angle in radians instead of degrees. The definition of the radian is:



arc length = s
$\theta_{radians} = S/R$

Thus $\theta_{radians} R$ = arc length = S

Exercises

1. What is the arc length if R = 2 m and $\theta_{radians} = 0.2$?

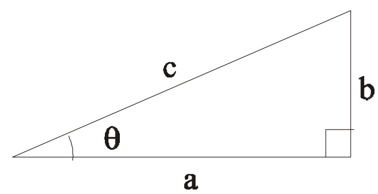
2. How many radians are there in 360^o? In 180^o?

Trigonometry

Trigonometry is essential to PHY 2053-2054. While it is sometimes used otherwise, the most common use in this course is in quantitative treatment of vectors, which will be taught in the lecture-recitation part of the course.

$$\sin q = \frac{b}{c}, \cos q = \frac{a}{c}$$

$$\tan q = \frac{b}{a} = \frac{\frac{b}{c}}{\frac{a}{c}} = \frac{\sin q}{\cos q}$$



The definitions of the trigonometric functions are:

$$\sin q^2 + \cos q^2 = \frac{b^2}{c^2} + \frac{a^2}{c^2} = \frac{b^2 + a^2}{c^2} = \frac{c^2}{c^2} = 1$$

A useful trigonometric identity (about the only one used in Physics 2053-54) is: (the Pythagorean theorem)

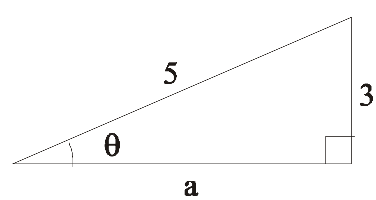
Some special values of trigonometric functions commonly used in physics courses:

$30^o = \pi/6$ radians
$\sin 30^o = 1/2$; $\cos 30^o = \sqrt{3}/2$; $\tan 30^o = 1/\sqrt{3}$
$\alpha = 45^o = \pi/4$ radians
$\sin 45^o = 1/\sqrt{2} = \cos 45^o$; $\tan 45^o = 1$

Example

If $\sin\theta = \frac{3}{5}$ find $\cos\theta$ and $\tan\theta$ and θ.

By definition of the sin:



(Of course the sides could be 6 and 10, or any other numbers such that the opposite/hypotenuse $= \frac{3}{5}$).

Exercises

1. If $\cos\theta = \frac{2}{5}$ find $\sin\theta$ and $\tan\theta$.

2. Using your calculator or tables, find the sin, cos, and tan of 12^{o}.

3. From a point on the ground 96 ft. from a tower, the angle of elevation (the angle the top of the tower makes with the ground 96 ft. from the tower) of the top of the tower is 48.6^{o}. Sketch this situation and find the height of the tower.

4. From the top of a building 100 feet high over looking the ocean a man sees a boat sailing directly toward him. If the angle between his line of sight and the horizontal changes from 25^{o} to 40^{o} during his observation, find the distance the boat travels. First make a careful sketch or the situation.

5. Find the angle the line or sight of the sun makes with the horizontal if the shadow cast by a 5.0 ft. tall girl is 4.0 ft. long.

6. Find sin 233^{o}. Sketch this.

7. Sketch $\sin\theta$ and $\cos\theta$ as a function of θ for = 0 to 720^{o}. Put a radian angle scale on the θ axis as well as degrees. (This may be a careful freehand sketch).

8. If $y = A\cos(n\pi)$ find the values of n for which $y = 0$.

Summary of math review

Some useful Algebraic Rules.

If a = b then a + c = b + c and ac = bc.

a + b = b + a

(a + b) c = ac + bc

- (-a) = a

$$If\ \frac{a}{f} = \frac{c}{d},then\ \ ad = fc.$$

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$$

$$\frac{a}{b} \bullet \frac{c}{d} = \frac{ac}{bd}$$

The same number (symbol) can be added to or subtracted from both sides of an equation (which always involves an equality sign) without changing the validity of the equation.

ax + by = c(x - y) ; ax + by + z = c(x - y) + z
; ax + by - 2z = c(x - y) - 2z

Both sides of an equation may be multiplied or divided by the same number or symbol without changing the validity or the equation.

ax + by = c(x +y) ; z(ax + by) = axz + bzy = zc(x+ y) = zcx + czy

$$\frac{ax+by}{z} = \frac{c}{z}\ (x+y)$$

exponents:

$a^m\ a^n = a^{m+n}$
$(a^m)^n = a^{mn}$
$(ab)^n = a^n b^n$
$(a/b)^m = a^m/b^m = a^m b^{-m}$

multiplication:

$(x + y)\ (x - y) = x^2 - y^2$
$(x + y)^2 = x^2 + 2xy + y^2$

$$(ax + by)(cx + dy) = acx^2 + adxy + bcxy + bdy^2$$
$$= acx^2 + (ad + bc)\,xy + bdy^2$$

The roots of $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 \quad 4ac}}{2a}$$

Logarithms: $\log_a xy = \log_a x + \log_a y$
$\log_a x/y = \log_a x - \log_a y$
$\log_a x^n = n \log_a x$

Geometric relations

right triangle A = 1/2 bh (b = base; h = height, A = area)

rectangle A = bh

square $A = b^2$

circle Circumference = c = $2\pi r$ (r = radius)
$A = \pi r^2$

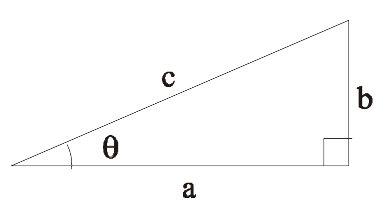
sphere surface area = $4\pi r^2$ (r = radius)
$V = \frac{4}{3}\pi r^3$, V = volume

Trigonometry

$$c^2 = a^2 + b^2$$

$$\sin q = \frac{b}{c};\ \cos q = \frac{a}{c};\ \tan q = \frac{b}{c} = \frac{\sin q}{\cos q}$$

$$\sin q^2 + \cos q^2 = \frac{b^2}{c^2} + \frac{a^2}{c^2} = \frac{b^2 + a^2}{c^2} = \frac{c^2}{c^2} = 1$$



This completes the math review. It is not completely comprehensive, but if you have carefully worked through and understood the examples and exercises you should have a good grasp of most of the mathematical tools used in Physics 2053-2054. Good Luck!

Laboratory Work

As you study the various laws and principles of physics it is important for you to keep in mind that these laws and principles are valid because they agree with the results of experiment. Since experiments are never exact, it is the task of the experimenter to obtain not only the best data possible, but also to assess in some manner how accurate the resulting data are. Without an assessment of experimental error, there is no way to know whether any deviation of experimental results from those predicted by theory is significant.

Obviously, a first laboratory course in physics is not where stringent tests of physical laws are made. Instead such a laboratory is designed to introduce you to the world of experimental physics. Its aim is to help you develop familiarity with the experimental method and to gain experience in the actual handling of laboratory apparatus.

By its nature, experimental work requires patience, care, and a willingness to go back and redo measurements when it is discovered that poor or improper procedure has been used. The old adage that "when all else fails read the instructions" applies well to this lab. The proper approach is to read the experimental write-up before coming to lab and then to re-read each section carefully in the lab before doing the work in that section. There is no excuse for going through an experimental procedure only to find that if you had read a few sentences further in the write-up you could have avoided costly mistakes. The equipment in this lab, though simple, will yield unambiguous results if some care is exercised. Part of your training is to learn how to exercise such care.